Other results similar to (17) and (20) are readily obtained. All follow from the fact that successive differentiation or integration of a power-series introduces into the nth term a factor involving factorial expressions in n.

4. Tweedie has derived[1] numerous identities among the Stirling numbers by the use of the expansion (1′). However, the following relation is not, I believe, included in any he has given. By (1′) and (4),

$$(23)\qquad (x-1)^m = \Gamma_2^{m-1}(x-1) + \Gamma_3^{m-2}(x-1)(x-2) + \cdots + \Gamma_{m+1}^0(x-1)\cdots(x-m),$$

and

$$(24)\quad (x-1)(x-2)\cdots(x-r) = C_{r+1}^0 x^r - C_{r+1}^1 x^{r-1} + \cdots (-)^r C_{r+1}^r.$$

Substituting for the factorials in (23) their equals in (24), and equating the coefficients of x^n on the two sides of the resulting equation, we have:

$$(-1)^{m-n}\binom{m}{n} = \Gamma_{n+1}^{m-n} C_{n+1}^{0} - \Gamma_{n+2}^{m-n-1} C_{n+2}^{1} + \cdots (-1)^{m-n} \Gamma_{m+1}^{0} C_{m+1}^{m-n},$$

i.e., dividing by $(-1)^{m-n}$, and replacing m by $m-1$ and n by $n-1$,

$$(25)\quad \Gamma_m^0 C_m^{m-n} - \Gamma_{m-1}^1 C_{m-1}^{m-n-1} + \cdots (-1)^{m-n} \Gamma_n^{m-n} C_n^0 = \binom{m-1}{n-1},\ m > n > 0.$$

In particular, for $n=1$:

$$(26)\qquad (m-1)!\Gamma_m^0 - (m-2)!\Gamma_{m-1}^1 + \cdots (-1)^{m-2} 1!\Gamma_2^{m-2} = 1,\ m > 1.$$

I take this occasion to note an error in Ginsburg's article. The values he gives for ${}_nS_r'$ (i.e., the sum of the r-products of the first n integers, repetitions being included), are actually those of ${}_{n-1}S_r'$. In fact, ${}_nS_r' = \Gamma_{n}{}^{r}{}_{+1}$, while ${}_nS_r$ (i.e., the corresponding sum without repetitions) $= C_{n+1}^r$. The latter values are given correctly.

RECENT PUBLICATIONS

EDITED BY R. A. JOHNSON, Brooklyn College of the City of New York

All books for review should be sent directly to the editor of this department, at Brooklyn College, 66 Court Street, Brooklyn, N. Y., and not to any of the other editors or officers of the Association.

REVIEWS

Science and Sanity. An Introduction to Non-Aristotelian Systems and General Semantics. By Alfred Korzybski. Lancaster, The Science Printing Company, 1933. xx+798 pages. $7.00; with discount, $5.50.

In the limits of a short notice it is impossible even to indicate the wealth of material in Korzybski's introduction to general semantics,[2] ranging as it does

[1] Op. cit., pp. 11–14.

[2] From σημαίνειν "to signify," "to mean."

from a general discussion of "structure," through "the non-Aristotelian language called mathematics" and "the foundations of psychophysiology" to "the sematics of the differential calculus" and "the structure of matter"; so we shall attempt merely to indicate a few of the high spots which readers of this MONTHLY will find of particular interest. Much of the book represents pioneering work by the author, who insists that the greatest value of his new approach is in its experimental and practical possibilities for sane behaviour rather than its philosophical importance. For a more adequate review the reader is referred to an article by Professor Keyser.[1]

Mathematical readers will probably best make their way into the book by reading first Supplement III, pp. 747–761, where Korzybski orients part of his work with respect to the four leading schools of current mathematical thought, namely the logistic (Peano, Whitehead, Russell), the axiomatic (Hilbert), the intuitional (Brouwer, Weyl), and the Polish non-Aristotelian schools (Łucasiewicz, Tarski, Leśniewski, Skarżeński, Chwistek). To these Korzybski adds a fifth school: "The average prevalent mathematical technician, who does not realize that he belongs to the numerically large class which may be called the 'Christian Science' school of mathematics, which proceeds by faith and disregards entirely any problems of the epistemological foundations of their supposed 'scientific' activities." This sounds rather unkind. But is it wholly undeserved? Let him that is without sin among us cast the first stone.

Korzybski's system is definitely non-Aristotelian in several respects, only one of which can be noted here. Korzybski rejects outright Aristotle's first law, the so-called law of identity, which is quoted in Jevons' form: "Whatever is, is." Another statement of the law is "*A* is *A*." Readers familiar with Russell's "theory of types" will recognize an isomorphism between it and Korzybski's sharply clear account of the different "levels of abstraction" by which mathematicians and others verbalize brute objects, like bricks, at the *unspeakable* level, into symbols, like the word b-r-i-c-k-s, which in their turn are "named," or otherwise raised to a higher level of abstraction, and so on, apparently indefinitely. Blunders in reasoning and common, plain thinking multiply when the levels are confused, and the "is" of identity appears as the most prolific source of such confusions. From this point of view it would seem that the famous "axiom of reducibility" could never even come into sight. Whether or not this is so, it will be clear to any reader who takes the pains to understand Korzybski's position that the author has raised an issue of the greatest interest to all those who seek to understand the foundations of mathematics.[2]

[1] *Scripta Mathematica*, vol. 2 (1934), pp. 247–260.

[2] Professor Keyser (loc. cit., p. 246) remarks that Korzybski, "in proposing to eliminate the 'is of identity' completely from all linguistic structure, has gone far beyond all other critics, Aristotle included. In fairness, however, to Aristotle, it must be said that he did not fail to note that peculiar sense, among the several senses, of the term 'is' and did not fail to indicate the danger of employing it uncritically." Aristotle, no less than the devil, must be given his due. But it seems to the reviewer that Professor Keyser, as shown by the quotations from Aristotle on p. 254 of his article, gives Aristotle considerably more than his due. Korzybski's rejection, in its relation to

Mathematicians will get a good idea of Korzybski's program by reading (pp. 93–94) the explicit statements of certain of the things which Korzybski either accepts or rejects. Among the acceptances are relations, structure, and order (concepts undefined in the system). Readers acquainted with attempts to define these concepts in mathematics[1] may be willing to grant the author that these three are as well left undefined at present. However, all of them, and in particular "structure," which is all-important for Korzybski's position, are sufficiently explained—explanation of course is not definition by a set of postulates.

General semantics itself is described as the science of significant behaviour. This may remind some of Clive Bell's definition of art as "significant form," and the experience of Bell's readers in learning from his book on the subject that "significant form" is nothing more nor less than "significant form." Was there not some difficulty in *Principia Mathematica* over the question of "significant" statements— the presumed insight on the part of the reader which would enable him to judge whether a given statement was "significant" or just a "meaningless" jumble of words or symbols? Anyhow, Wittgenstein appears to have seen through this particular difficulty, and to have disposed of it (temporarily) with his decree that "mathematical truths" shall be analytic. However, it will probably be agreed by all readers that Korzybski has given them a detailed description of what he means by general semantics, and that he has illuminated mathematics by placing it as a detail, but a highly important one, in his vaster picture.

Another innovation is the wholesale exploitation of what Korzybski calls "non-elementalism," already classical in theoretical physics through the fusion of "space" and "time" into "space-time" by Einstein and Minkowski. Closely allied to this is the insistence upon the organism-as-a-whole point of view of certain biologists, which Korzybski also exploits. Thus (p. 30), "I must construct a non-elementalistic language in which 'senses' and 'mind,' · · · are no longer to be verbally split, because a language in which they *are* split is not similar in structure to the known empirical facts · · · ." It may be recalled in passing that Whitehead, some years ago, raised similar objections to what he called "bifurcation" theories of nature. How these extremely general and far-reaching ideas are applied to mathematics and to mathematical physics, must be seen in the book itself.

Although it is a minor point in the sweep of the general development, there were some things in the chapter on "linearity" (pp. 603–614) which the reviewer found difficult to understand. Thus it is stated (p. 613) that "*approximation* · · · is strictly connected with *linearity* or *additivity*." From its context,

mathematics, is of a different kind from Aristotle's. It is difficult (at least for the reviewer) to see how Aristotle's rather naïve distinctions between the "senses borne by the term 'Sameness' " could apply to Korzybski's *levels of abstraction*, or to Russell's *types*. It is not clear, from the quotations (which see), that Aristotle's position is relevant for Korzybski's.

[1] For relations, the Whitehead-Russell definition; for order, projective geometry, where it is assumed that *abc* and *acb* are distinguishable and distinct orders of the three letters involved; for structure, any of the attempts to define it in fairly recent work (including some of Russell's).

this seems to imply that in *all* kinds of approximative work in mathematics it is sufficient to consider only terms of order not higher than the first. This is contradicted by so simple a thing as the indicatrix. If by "strictly connected" is meant "semantically connected," then again the statement seems to say too much. There is nothing sacrosanct about the linearity of certain differential equations (and hence the additivity of their solutions) that makes most of mathematical physics as we know it a possibility; a more competent generation may find that linearity is a gratuitous concession to present mathematical disabilities. It has been conjectured (although possibly not in print) by Einstein that some of our failures to give a coherent (="semantic," in Korzybski's sense) account of some physical phenomena may be rooted in the traditional demand for linearity. This is not the place to go into the history of this demand, but a consideration of it from Huyghens to the present might show that it is on a par with any other postulate, sufficient so long as it serves, but not necessarily "significant" at any time or in any place (or in any "time-place," to be non-elementalistic). The existence of doubts as to its sufficiency seem to indicate that a revision of its "semantic" status is about due. But, as already stated, this is a minor point, and we shall ignore others of a similar character, since, to dwell upon them, would only give a false impression of the book.

For students of mathematics, probably the most illuminating of the many new points of view in the book will be those which re-value mathematics in the light of human experience as a whole. Korzybski emphasizes that mathematics and mathematical physics have succeeded better in their self-appointed tasks than some other human enterprises because the *structure* of both is more closely patterned than is that of any other "language" to the thing which is to be undertaken. To indicate the basis for this claim we must refer to the book itself. Further, mathematics is here brought down from the celestial void of pure disembodied thought. Like Brouwer, Korzybski regards mathematics as a form of human behaviour or, differently expressed, as a social activity of human beings. Unlike his predecessors, Korzybski backs his claim with a mass of evidence drawn from practically the entire range of science—including the biological sciences—such as has not been assembled in any one place before. Mathematicians will find their estimates of their activities both inflated and deflated by a reading of this remarkable book.

E. T. BELL

Einführung in die Differentialrechnung und Integralrechnung. By Edmund Landau, Groningen, P. Noordhoff, 1934. 368 pages.

This treatise on the calculus is based on lecture courses given repeatedly by Professor Landau at Göttingen. It is not a faithful reproduction of his lectures. For instance, Professor Landau devoted considerable time, in his courses, to geometric applications. Not wishing to presuppose a knowledge of the foundations of geometry, and being anxious at the same time that his book possess complete rigor, Professor Landau restricted himself to purely analytic aspects of the calculus.

This work is thus not to be compared with ordinary calculus texts. Except for the fact that it leaves the general theory of point sets almost untouched, it may be regarded as an exposition of the theory of functions of a real variable. As a reference work in courses on the real variable, it will be found very valuable.

There is presupposed, on the part of the reader, such an acquaintance with the properties of real numbers as can be gained from Landau's recent monograph *Grundlagen der Analysis*.

In the first part, which deals with the differential calculus, the general topics studied are sequences, functions and continuity, derivatives, infinite series, the Taylor expansion, functions of two variables and implicit functions. There are chapters on the elementary transcendental functions. Special topics of interest are the fundamental theorem of algebra and the decomposition of rational functions into partial fractions. The spirit of the work is shown by such details as the proof of the representability of a continuous function as a limit of polynomials, and the proof, given almost immediately after the definition of derivative, of the existence of functions which lack a derivative everywhere.

In the second part, on the integral calculus, the general topics are the indefinite integral, the integral as a limit of a sum, the integration of infinite series and improper integrals. The integrals of rational functions are examined in detail. A brief treatment is given of the gamma function. The book closes with a chapter on Fourier series, in which an expansion theorem sufficient for the ordinary applications is obtained.

Edmund Landau is a great mathematician and a great expositor, of whom the whole mathematical world may be justly proud. His writings perpetuate brilliantly the traditions of Gauss and of Weierstrass. This latest book will survive, in its crystalline beauty, an inspiration to students of all nations, long after the chauvinistic anthropological twaddle which has assailed mathematical ears in recent months has passed on to its appropriate limbo.

J. F. Ritt

MATHEMATICS CLUBS

Edited by F. M. Weida, The George Washington University, Washington, D. C.

All reports of club activities, suggestions and topics for club programs should be sent to F. M. Weida, The George Washington University, Washington, D. C. All manuscripts should be typewritten with double spacing, and with margins at least one inch wide.

Club Activities

1933–1934

THE PI MU EPSILON MATHEMATICAL FRATERNITY

Pi Mu Epsilon is an academic fraternity in institutions of university grade. Its primary aim is